DAMTP 93-46

Charged Particle Scattering From Electroweak and Semi-local Strings

A.C.Davis[*]and A.P.Martin,

DAMTP,

Cambridge University,

Silver St.,

Cambridge

CB3 9EW

U.K.

N.Ganoulis,

Physics Division,

School of Technology,

Thessaloniki,

Greece.

February 1, 2008

Abstract

The scattering of a charged fermion from an electroweak or semi-local string is investigated and a full solution obtained for both massive and massless cases. For the former, with fractional string flux, there is a helicity conserving and helicity-flip cross-section, of equal magnitude and of a modified Aharonov-Bohm form: for integer flux the strong interaction cross-section is suppressed by a logarithmic term. The results also apply for GUT cosmic strings and chiral fermions.

[*]and King's College, Cambridge

1 Introduction

The existence of string defects in models displaying spontaneous symmetry-breaking with topologically non-trivial vacuum manifolds has been known for some time [1]. Many grand unified theories, for example, possess local string solutions and the axion model of Peccei and Quinn [2] has a global string solution. The simplest model of a local string is the Nielsen-Olesen vortex [3]. For fractional string flux the dominant scattering for a charged particle off such a string is Aharonov-Bohm [4][5], whilst if the string flux is integer, one obtains the scattering cross-section of Everett [6]. Recently, a wider class of string defects, the semi-local [7][8] and the electroweak strings [9] have been discovered in theories where the first homotopy group of the vacuum manifold is topologically trivial. These, the embedded defects, are at best metastable. Since this new class of string defects has special features, it is interesting to see if they also exhibit Aharonov-Bohm scattering. This is the aim of this paper.

The Aharonov-Bohm scattering of fermions off electroweak strings has been studied by one of us before [10]. There it was found that the scattering cross-section violated helicity, and went to zero in the limit of zero mass particles. However, due to the complexity of the problem a full solution was not obtained and the dominant mode approximation used. In this paper we are able to go beyond this approximation to obtain a fuller scattering solution using the 'top-hat' model of the string-core, discussed in [5]. There it was shown that the scattering cross-section was insensitive to the core-model, and the simple 'top-hat' model gave the same result as the more sophisticated one. Using this method we are able to clarify the results of [10] and show that, in the massive case, for fractional flux there are both helicity conserving and helicity-flip Aharonov-Bohm scattering cross-sections with the electroweak string. The helicity violating cross-section is seen to go to zero in the limit of zero mass. We also show that there is an analogue of the Everett cross-section in the case of integer flux and massive particles.. This is also the case for semi-local strings. Hence, for both electroweak and semi-local strings with fractional flux the elastic-scattering cross-section is a strong interaction cross-section, independent of the string radius. For integer flux there is only a logarithmic suppression of the strong interaction cross-section. Thus, if such strings are metastable they will interact with the surrounding plasma in an analogous way to local topological strings [11].

The plan of this paper is as follows:- In section 2 we review the semi-local and electroweak strings. We also recap the work of [10] for scattering from electroweak strings. In section 3 we calculate the scattering amplitudes of fermions in interaction with electroweak strings in both massive and massless cases, making use of the 'top-hat' core model and in section 4 we obtain the corresponding cross-sections in the cases of integer and non-integer flux. This method is also applied to the case of fermions scattering off semi-local strings in section 5, our conclusions on this work being contained in section 6.

2 Semi-local and Electroweak Strings

The general requirement for the formation of topological string defects is that the vacuum manifold is multiply connected. The simplest model of this is the U(1) Nielsen-Olesen vortex [3]. Recently, however, Vachaspati and Achúcarro [7] demonstrated that if *both* global *and* local symmetries are present in a model then it is possible to have stable string solutions even if the vacuum manifold is simply connected. The fact that they share properties with both local and global defects led to them being dubbed semi-local strings.

The string investigated in [7] arises in an extension of the Abelian Higgs model where the complex scalar field is replaced by an SU(2) doublet, Φ, such that the action is invariant under G=$\mathrm{SU}(2)_g \times \mathrm{U}(1)_l$ where g and l denote global and local symmetries respectively. When Φ condenses, breaking G to H=$\mathrm{U}(1)_l$ the corresponding vacuum manifold is the 3-sphere, S^3. This is clearly simply connected and the formation of string defects would not normally be expected. If, however, we now consider the gauge component of the symmetries alone, then it is seen that the vacuum manifold is a circle, and, hence, not simply connected. This suggests the existence of string solutions. The way to reconcile this apparent paradox is to consider the change in energy in passing between such solutions.

For each point on the vacuum manifold, the $\mathrm{U}(1)_l$ transformation generates a circle around the three-sphere; each of these circular paths, in turn, corresponds to a string solution. Hence, we can think of the vacuum manifold as comprising of the set of all possible string solutions. To move to another point on the manifold along one of these circular paths involves a gauge transformation and so expends no energy. However, to leave such a path involves a global transformation and thus requires a change in energy. These energy barriers are what enable string solutions to exist on a topologically trivial manifold, since to contract a loop to a point (the normal way of removing such defects) always involves a cost in energy. It has been shown that the stability of semi-local strings is parameter dependent, depending on the ratio of the masses of the Higgs and vector particles [8].

Now, the extended Abelian-Higgs model that yields the semi-local string is none other than the Electroweak model with zero SU(2) charge. This led to the postulation of non-topological, yet metastable, strings in the Electroweak theory [9]. Indeed, such a solution is found, and bears a predictable likeness to the semi-local case.The existence of such strings is closely linked to that of the semi-local string. Whilst the string solution is unstable in the minimal electroweak theory [12], extensions may possess stable string solutions.

Consider the string solution in the standard electroweak theory with fermionic interactions, the relevant part of the Lagrangian being

$$\mathcal{L} = i\bar{L}\gamma^\mu D_\mu L + i\bar{e}_R\gamma^\mu D_\mu e_R - f_e(\bar{L}e_R\Phi + \Phi^\dagger\bar{e}_R L) \tag{1}$$

where $\bar{L} = (\bar{\nu}, \bar{e}_L)$, f_e is the Yukawa coupling, Φ is the usual Higgs field and the covariant derivative is given by

$$D_\mu = \partial_\mu + \frac{i\alpha\gamma}{2} Z_\mu \tag{2}$$

where $\gamma = e/(\sin(\theta_W)\cos(\theta_W))$ and θ_W is the Weinberg angle. The Z-coupling, α, has the form

$$\alpha = -2(T_3 - Q\sin^2\theta_W) \tag{3}$$

where T_3 is weak isospin and Q is electric charge. This clearly varies according to the field to which it is coupled. In particular we note that for electrons and down quarks,

$$\alpha_L = \alpha_R + 1 \tag{4}$$

so we have a marked asymmetry between left and right fields[1].

In [10], when considering the scattering of fermions off an electroweak string, use was made of the solution derived in [9], where a U(1) Nielsen-Olesen string is embedded in SU(2)×U(1). More explicitly

$$\begin{aligned} \Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} &= f(r)e^{i\phi}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ Z_\phi = -\tfrac{v(r)}{r}, \qquad & Z_r = 0, \\ W = 0, \qquad & A = 0, \end{aligned} \tag{5}$$

where Z and W are the usual gauge bosons, and A is the photon field. The functions f and v are functions of r only: they are the usual Nielsen-Olesen solutions, found by solving the appropriate field equations and have approximate profiles

$$\begin{aligned} f(r) &= \begin{cases} \frac{\eta}{\sqrt{2}} & r \geq R \\ \frac{\eta}{\sqrt{2}}\left(\frac{r}{R}\right) & r < R \end{cases} \\ v(r) &= \begin{cases} \frac{2}{\gamma} & r \geq R \\ \frac{2}{\gamma}\left(\frac{r}{R}\right)^2 & r < R \end{cases} \end{aligned}$$

We note that since Z and Φ are independent of z the equations of motion will be, essentially, 2+1 dimensional Dirac equations.

Considering the case of electrons only for the time being, if we set $m = f_e\eta/\sqrt{2}$ and $\partial_t = -i\omega$ where ω is the energy of the electron, then taking the usual Dirac representation and writing $e_L = (0, \psi)$, $e_R = (\chi, 0)$ we obtain the following equations of motion for ψ and χ;

$$\begin{aligned} \omega\chi + i\sigma^j D_j^R\chi - f_e f e^{-i\phi}\psi &= 0, \\ \omega\psi - i\sigma^j D_j^L\psi - f_e f e^{i\phi}\chi &= 0. \end{aligned} \tag{6}$$

Note the phases $e^{\pm i\phi}$ and the coupling of ψ to χ via the mass term. The effects of these, plus the different couplings to the Z, manifest themselves in a non-zero amplitude for helicity flip.

To see that helicity is not conserved consider the following. From (6) it is easy to write down the Hamiltonian for the system using canonical momenta; $\underline{\sigma}.\underline{\pi} = -i\sigma^j D_j$

$$H = \begin{pmatrix} -\underline{\sigma}.\underline{\pi}_R & f_e f e^{-i\phi} \\ f_e f e^{i\phi} & \underline{\sigma}.\underline{\pi}_L \end{pmatrix}. \tag{7}$$

The helicity operator is defined as

$$\underline{\Sigma}.\underline{\Pi} = \begin{pmatrix} \underline{\sigma}.\underline{\pi}_R & 0 \\ 0 & \underline{\sigma}.\underline{\pi}_L \end{pmatrix}. \tag{8}$$

[1]For up quarks the relationship is reversed with $\alpha_L = \alpha_R - 1$, but we still have an asymmetry between left and right fields

If helicity was conserved one would expect the commutator of the Hamiltonian with the helicity operator to be zero. However, when calculated, one finds that

$$[H, \underline{\Sigma}.\underline{\Pi}] = i f_e \left(\begin{array}{cc} 0 & \sigma^j (D^j \phi^0)^* \\ \sigma^j D^j \phi^0 & 0 \end{array} \right), \tag{9}$$

which is non-zero inside the string core. This non-conservation of helicity differs from the usual Aharonov-Bohm scattering off a thin solenoid of magnetic flux only. It is noted, however, that if we take the massless case where $f_e = 0$ and we have no coupling betwen the ψ and χ, then $[H, \underline{\Sigma}.\underline{\Pi}] = 0$ and helicity will be conserved. Hence, we expect the cross-sections for the massless and massive cases to differ.

Considering the case of an incoming plane wave of positive helicity, it can be demonstrated [10] that one mode tends to dominate the scattering, the cross-sections for this mode being identical for positive and negative helicity scattered states, such that, to leading order,

$$\left.\frac{d\sigma}{d\theta}\right|_{\pm} \sim \frac{1}{k} \left(\frac{\omega - k}{2\omega} \right)^2 \sin^2 (\pi \alpha_R) \tag{10}$$

where k is the momentum of the electron. This is, however, only a partial result, so a natural next step is to try and obtain a fuller result by means of a simpler profile for Z and Φ.

3 Scattering from the Electroweak String

Our method follows those of [10] and [5], though we adopt the simpler "top-hat" model in an attempt to get a fuller result:

$$f(r) = \begin{cases} 0 & , \quad r < R \\ \eta/\sqrt{2} & , \quad r > R \end{cases} \tag{11}$$

$$v(r) = \begin{cases} 0 & , \quad r < R \\ 2/\gamma & , \quad r > R \end{cases} \tag{12}$$

The equations of motion are the same as in (6), up to the altered gauge fields and covariant derivatives. We note that although the χ and ψ are coupled via the mass term, using the above profile, there is no mass inside the core and hence no coupling.

We try the usual mode decomposition

$$\chi(r,\phi) = \sum_{l=-\infty}^{\infty} \begin{pmatrix} \chi_1^l(r) \\ i\chi_2^l(r)e^{i\phi} \end{pmatrix} e^{il\phi}, \quad \psi(r,\phi) = \sum_{l=-\infty}^{\infty} \begin{pmatrix} \psi_1^l(r) \\ i\psi_2^l(r)e^{i\phi} \end{pmatrix} e^{i(l+1)\phi}. \tag{13}$$

Making use of

$$\sigma^j D_j = \begin{pmatrix} 0 & e^{-i\phi}(D_r - iD_\phi) \\ e^{i\phi}(D_r + iD_\phi) & 0 \end{pmatrix} \tag{14}$$

we now substitute (13) and (14) into (6) to obtain

$$\begin{aligned}
\omega\chi_2^l &+ (\tfrac{d}{dr} - \tfrac{l}{r} + \tfrac{\alpha_R \gamma v}{2r})\chi_1^l - f_e f \psi_2^l = 0, \\
\omega\chi_1^l &- (\tfrac{d}{dr} + \tfrac{l+1}{r} - \tfrac{\alpha_R \gamma v}{2r})\chi_2^l - f_e f \psi_1^l = 0, \\
\omega\psi_2^l &- (\tfrac{d}{dr} - \tfrac{l+1}{r} + \tfrac{\alpha_L \gamma v}{2r})\psi_1^l - f_e f \chi_2^l = 0, \\
\omega\psi_1^l &+ (\tfrac{d}{dr} + \tfrac{l+2}{r} - \tfrac{\alpha_L \gamma v}{2r})\psi_2^l - f_e f \chi_1^l = 0.
\end{aligned} \tag{15}$$

We also insist that our solutions are eigenfunctions of the helicity operator, which implies that

$$\begin{aligned}
\underline{\sigma}.\underline{\pi}_R \chi &= \pm k\chi, \\
\underline{\sigma}.\underline{\pi}_L \psi &= \pm k\psi,
\end{aligned} \tag{16}$$

where k is the momentum, and $+(-)$ corresponds to positive(negative) helicity. On substitution in (15) this yields

$$\begin{aligned}
(\omega \mp k)\chi &= f_e f \psi, \\
(\omega \pm k)\psi &= f_e f \chi,
\end{aligned} \tag{17}$$

so giving us a relation between χ and ψ.

We now need to consider solutions of the Dirac equation inside and outside the string core.

(a) Internal solution: $r < R$

Taking the profiles described earlier we have $f = v = 0$ for $r < R$, so our equations of motion (15) reduce to

$$\begin{aligned}
\omega\chi_2^l \quad &+ \quad (\tfrac{d}{dr} - \tfrac{l}{r})\chi_1^l \quad = \quad 0, \\
\omega\chi_1^l \quad &- \quad (\tfrac{d}{dr} + \tfrac{l+1}{r})\chi_2^l \quad = \quad 0, \\
\omega\psi_2^l \quad &- \quad (\tfrac{d}{dr} - \tfrac{l+1}{r})\psi_1^l \quad = \quad 0, \\
\omega\psi_1^l \quad &+ \quad (\tfrac{d}{dr} + \tfrac{l+2}{r})\psi_2^l \quad = \quad 0.
\end{aligned} \tag{18}$$

Consider first the χ. Combining the top two equations of (18) and setting $z = \omega r$ we obtain

$$\tfrac{1}{z}\tfrac{d}{dz}(z\tfrac{d}{dz})\chi_1^l \quad + \quad (\tfrac{z^2-l^2}{z^2})\chi_1^l \quad = \quad 0, \tag{19}$$

which is easily recognised as Bessels equation of order l. Hence, by square integrability and regularity at the origin, the internal solution is

$$\chi_1^l = c_l J_l(\omega r). \tag{20}$$

Now, χ_1^l and χ_2^l are coupled via (18) so, making use of the Bessel function relations[2]

$$\tfrac{d}{dz}J_\mu(z) \quad = \quad \tfrac{1}{2}(J_{\mu-1}(z) - J_{\mu+1}(z)) \quad , \quad \tfrac{\mu}{z}J_\mu(z) \quad = \quad \tfrac{1}{2}(J_{\mu-1}(z) + J_{\mu+1}(z)), \tag{21}$$

it is easy to show that $\chi_2^l = c_l J_{l+1}(\omega r)$.

Inside the core, however, $f = 0$ so, unfortunately, (17) gives us no information about a link between χ and ψ. From (18), though, it is easy to see that ψ_1^l and ψ_2^l will satisfy Bessel equations of order $(l+1)$ and $(l+2)$ respectively, so our internal solution is

$$\begin{pmatrix} \chi \\ \psi \end{pmatrix} = \sum_{l=-\infty}^{\infty} \begin{pmatrix} c_l J_l(\omega r) \\ ic_l J_{l+1}(\omega r)e^{i\phi} \\ d_l J_{l+1}(\omega r)e^{i\phi} \\ id_l J_{l+2}(\omega r)e^{2i\phi} \end{pmatrix} e^{il\phi} \tag{22}$$

where c_l and d_l are independent.

[2]These actually hold for Bessel functions of the first, second or third kind.

(b) External solution: $r > R$

For large r, we now take $f(r) = \frac{\eta}{\sqrt{2}}$ and $\frac{v(r)}{r} = \frac{2}{\gamma r}$, so that, defining $m = \frac{f_e \eta}{\sqrt{2}}$, our equations of motion become

$$\begin{aligned}
(\tfrac{d}{dr} - \tfrac{l}{r} + \tfrac{\alpha_R \gamma v}{2r})\chi_1^l &+ \omega\chi_2^l - m\psi_2^l = 0, \\
(\tfrac{d}{dr} + \tfrac{l+1}{r} - \tfrac{\alpha_R \gamma v}{2r})\chi_2^l &- \omega\chi_1^l + m\psi_1^l = 0, \\
(\tfrac{d}{dr} - \tfrac{l+1}{r} + \tfrac{\alpha_L \gamma v}{2r})\psi_1^l &- \omega\psi_2^l + m\chi_2^l = 0, \\
(\tfrac{d}{dr} + \tfrac{l+2}{r} - \tfrac{\alpha_L \gamma v}{2r})\psi_2^l &+ \omega\psi_1^l - m\chi_1^l = 0.
\end{aligned} \tag{23}$$

Setting

$$L = \begin{pmatrix} -l & 0 & 0 & 0 \\ 0 & l+1 & 0 & 0 \\ 0 & 0 & -l-1 & 0 \\ 0 & 0 & 0 & l+2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$Q = \begin{pmatrix} \alpha_R & 0 & 0 & 0 \\ 0 & -\alpha_R & 0 & 0 \\ 0 & 0 & \alpha_L & 0 \\ 0 & 0 & 0 & -\alpha_L \end{pmatrix}, \qquad P = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$

and $w_l = (\chi_1^l, \chi_2^l, \psi_1^l, \psi_2^l)$ we can write (23) as

$$\frac{d}{dr} w_l + \frac{1}{r}(L+Q)w_l + (\omega C + mP)w_l = 0 \tag{24}$$

the asymptotic behaviour of which is decided by the eigenvalues of $(\omega C + mP)$; found to be $\pm ik$ as one would expect.

Defining $N = L + Q$ and $\nu = l - \alpha_R$, we can use our relation $\alpha_L - \alpha_R = 1$ to write N as

$$N = \begin{pmatrix} -\nu & 0 & 0 & 0 \\ 0 & \nu+1 & 0 & 0 \\ 0 & 0 & -\nu & 0 \\ 0 & 0 & 0 & \nu+1 \end{pmatrix}. \tag{25}$$

From this we see that (24) is the radial part of the free Dirac equation with shifted angular momentum. Taking the second derivative of (24), it can be shown that this reduces to

$$\frac{d^2}{dr^2} w_l + \frac{1}{r}\frac{d}{dr} w_l - \frac{N^2}{r^2} w_l + k^2 w_l = 0. \tag{26}$$

Thus $\chi_1^l, \chi_2^l, \psi_1^l, \psi_2^l$ obey independent Bessels equations of differing orders. By (17) however, for $r > R$,

$$\begin{aligned}
(\omega \mp k)\chi &= m\psi, \\
(\omega \pm k)\psi &= m\chi.
\end{aligned} \tag{27}$$

These together with (23) imply that

$$\chi_2^l = \mp \left(\frac{d}{dr} - \frac{\nu}{r} \right) \chi_1^l \tag{28}$$

where the upper (lower) sign corresponds to a positive (negative) helicity. Making use of our Bessel function relations (21) once more, we see, then, that the general solution for $r > R$ is

$$\begin{pmatrix} \chi \\ \psi \end{pmatrix} = \sum_{l=-\infty}^{\infty} \begin{pmatrix} Z_\nu(kr) \\ \pm i Z_{(\nu+1)}(kr) e^{i\phi} \\ B^\pm Z_\nu(kr) e^{i\phi} \\ \pm i B^\pm Z_{(\nu+1)}(kr) e^{2i\phi} \end{pmatrix} e^{il\phi} \tag{29}$$

where $B^\pm = m/(w \pm k)$ and Z_ν is a Bessel function of order ν. It is important to realise that when substituting in for the Bessel function, it is necessary to pay close attention to the sign of the order: for order ν we will have $Z_{(\nu+1)} = J_{\nu+1}, N_{\nu+1}$ whilst for $-\nu$ we will have $Z_{(\nu+1)} = -J_{-(\nu+1)}, -N_{-(\nu+1)}$. Also note that we have a k in the Bessel function arguments, as opposed to the ω in the internal solution. This arises because the mass outside the core is non-zero, so, as $\omega^2 = m^2 + k^2$, $\omega \neq k$.

(c) Asymptotic solution

We require, now, that our solution be an incoming planewave of, say, positive helicity, plus a scattered wave consisting of both positive *and* negative helicity states, since we know that helicity is not conserved in the case of finite mass. To apply this we need to equate our general external solution (6) with such an asymptotic form;

$$\sum_{l=-\infty}^{\infty}\begin{pmatrix}(-i)^{|l|}J_{|l|}e^{il\phi}\\ \pm i(-i)^{|l|}J_{\pm(l+1)}e^{i(l+1)\phi}\\ B^+(-i)^{|l|}J_{|l|}e^{i(l+1)\phi}\\ \pm iB^+(-i)^{|l|}J_{\pm(l+1)}e^{i(l+2)\phi}\end{pmatrix}+\frac{f_l e^{ikr}}{\sqrt{r}}\begin{pmatrix}e^{il\phi}\\ e^{i(l+1)\phi}\\ B^+e^{i(l+1)\phi}\\ B^+e^{i(l+2)\phi}\end{pmatrix}+\frac{g_l e^{ikr}}{\sqrt{r}}\begin{pmatrix}e^{il\phi}\\ -e^{i(l+1)\phi}\\ B^-e^{i(l+1)\phi}\\ -B^-e^{i(l+2)\phi}\end{pmatrix}$$

with $+(-)$ for $l>(<)0$. Here the first term is an incoming (and outgoing) plane wave of positive helicity, and the second and third terms are the positive and negative helicity components of the scattered wave respectively.

There are two cases to consider:

(i) $\boldsymbol{\nu}$ ≤-1, 0≤ $\boldsymbol{\nu}$: We take as our two independent solutions $Z^1_\nu = J_{|\nu|}$ and $Z^2_\nu = N_{|\nu|}$. Matching coefficients of $e^{il\phi}$ we obtain

$$\begin{array}{rcrcrcrcrcrcrcr} J_{|\nu|}a_l & + & N_{|\nu|}b_l & + & J_{|\nu|}A_l & + & N_{|\nu|}B_l & = & (-i)^{|l|}J_{|l|} & + & \frac{f_l e^{ikr}}{\sqrt{r}} & + & \frac{g_l e^{ikr}}{\sqrt{r}}\\ \pm J_{\pm(\nu+1)}a_l & \pm & N_{\pm(\nu+1)}b_l & \mp & J_{\pm(\nu+1)}A_l & \mp & N_{\pm(\nu+1)}B_l & = & \pm(-i)^{|l|}J_{\pm(l+1)} & - & \frac{if_l e^{ikr}}{\sqrt{r}} & + & \frac{ig_l e^{ikr}}{\sqrt{r}} \end{array}$$

Where $+(-)$ corresponds to $\nu>(<)0$ on the left-hand side, and $l>(<)0$ on the right-hand side. Making use of the large x form of the Bessel functions

$$J_\mu(x)\simeq\sqrt{\frac{2}{\pi x}}\cos\,(x-\frac{\mu\pi}{2}-\frac{\pi}{4}),\qquad N_\mu(x)\simeq\sqrt{\frac{2}{\pi x}}\sin\,(x-\frac{\mu\pi}{2}-\frac{\pi}{4}),\tag{30}$$

taking $0\geq\alpha_R>-1$ and eliminating a_l and A_l we find

$$\begin{aligned} f_l &= \frac{1}{\sqrt{2\pi ki}}\left[-2ie^{\mp\frac{i\pi\nu}{2}}b_l-(e^{\mp i\pi l}-e^{\mp i\pi\nu})\right],\\ g_l &= \frac{1}{\sqrt{2\pi ki}}\left[-2ie^{\mp\frac{i\pi\nu}{2}}B_l\right], \end{aligned}$$

where the $-(+)$ now just corresponds to $\nu>(<)0$.

(ii) -1< $\boldsymbol{\nu}$ <0: In this instance, it makes the algebra easier if we use $Z^1_\nu = J_\nu$ and $Z^2_\nu = J_{-\nu}$. We cannot do this in the previous case, however, because if ν is an integer, then J_ν and $J_{-\nu}$ are linearly dependent. Since $-1<\nu<0$, then $l=-1$ and $\alpha_R\neq 0$, so the choice is justified here. Matching coefficients of $e^{il\phi}$ we obtain

$$\begin{array}{rcrcrcrcrcrcrcr} J_\nu a_{-1} & + & J_{-\nu}b_{-1} & + & J_\nu A_{-1} & + & J_{-\nu}B_{-1} & = & -iJ_1 & + & \frac{f_{-1}e^{ikr}}{\sqrt{r}} & + & \frac{g_{-1}e^{ikr}}{\sqrt{r}}\\ J_{\nu+1}a_{-1} & - & J_{-\nu-1}b_{-1} & - & J_{\nu+1}A_{-1} & + & J_{-\nu-1}B_{-1} & = & iJ_0 & - & \frac{if_{-1}e^{ikr}}{\sqrt{r}} & + & \frac{ig_{-1}e^{ikr}}{\sqrt{r}} \end{array}$$

and making use of large x form of Bessel functions once more we can now eliminate a_l and A_l to obtain f_l and g_l in terms of b_l and B_l:

$$\begin{aligned} f_l &= \frac{1}{\sqrt{2\pi k i}}\left[e^{-\frac{i\pi\nu}{2}}(e^{i\pi\nu}-e^{-i\pi\nu})b_l+(1+e^{-i\pi\nu})\right], \\ g_l &= \frac{1}{\sqrt{2\pi k i}}\left[e^{-\frac{i\pi\nu}{2}}(e^{i\pi\nu}-e^{-i\pi\nu})B_l\right]. \end{aligned}$$

The asymptotic matching solutions also give us relationships between a_l and b_l, and, A_l and B_l. Considering the coefficients of e^{-ikr} for the two cases we find that for

$$\begin{array}{lll} \nu \geq 0, -1 \geq \nu: & a_l + i b_l &= (-i)^{\pm l} e^{\pm\frac{i\pi(l-\nu)}{2}}, \\ & A_l + iB_l &= 0, \\ \\ 0 > \nu > -1: & e^{\frac{i\pi\nu}{2}} a_l + e^{-\frac{i\pi\nu}{2}} b_l &= 1, \\ & e^{\frac{i\pi\nu}{2}} A_l + e^{-\frac{i\pi\nu}{2}} B_l &= 0, \end{array} \tag{31}$$

so we see that if either a_l or b_l dominates it will be of order 1, whilst A_l and B_l are always identical up to a phase.

(d) Matching at $r = R$

Now that we have solutions for inside and outside the core the obvious next step is to match them at $r = R$ to give a solution valid for all space. Clearly all of χ_1^l,χ_2^l,ψ_1^l,ψ_2^l will be continuous, though, due to the discontinuous distribution of the string flux, this will not be the case for their first derivatives. From (15), denoting the external (internal) solution by $+$ $(-)$, we see that

$$\begin{aligned} \left(\frac{d\chi_{1+}^l}{dr} - \frac{d\chi_{1-}^l}{dr}\right) &= -\frac{\alpha_R}{R}\chi_1^l + m\psi_2^l, \\ \left(\frac{d\chi_{2+}^l}{dr} - \frac{d\chi_{2-}^l}{dr}\right) &= \frac{\alpha_R}{R}\chi_2^l - m\psi_1^l, \\ \left(\frac{d\psi_{1+}^l}{dr} - \frac{d\psi_{1-}^l}{dr}\right) &= -\frac{\alpha_L}{R}\psi_1^l - m\chi_2^l, \\ \left(\frac{d\psi_{2+}^l}{dr} - \frac{d\psi_{2-}^l}{dr}\right) &= \frac{\alpha_L}{R}\psi_2^l + m\chi_1^l, \end{aligned}$$

all evaluated at $r = R$. Substituting in our external and internal solutions we find that the coefficients are related by

$$\begin{array}{rcrcrcl} S_1^+ a_l &+& S_2^+ b_l &+& (\lambda_l S_1^- + S_2^-)B_l &=& 0, \\ T_1^+ a_l &+& T_2^+ b_l &-& (\lambda_l T_1^- + T_2^-)B_l &=& 0, \\ B^+U_1^+ a_l &+& B^+U_2^+ b_l &+& B^-(\lambda_l U_1^- + U_2^-)B_l &=& 0, \\ B^+V_1^+ a_l &+& B^+V_2^+ b_l &-& B^-(\lambda_l V_1^- + V_2^-)B_l &=& 0, \end{array}$$

where

$$S_{1,2}^{\pm} = Z_\nu^{1,2\prime} - \left(\frac{J_l'}{J_l} - \frac{\alpha_R}{R} + mB^{\pm}\frac{J_{l+2}}{J_{l+1}}\right)Z_\nu^{1,2}$$

and we have similar expressions for T, U and V (the details are contained in the appendix to this chapter). We now have two cases to consider:

(i) $m \ll k$: Since $\omega^2 = m^2 + k^2$ we see that $B^+ = \frac{\omega - k}{m} \simeq \frac{k}{m}(1 + \frac{m^2}{2k^2}) - \frac{k}{m} \simeq 0$ whilst $B^- \simeq 2$. Hence, the last two matching solutions give that $B_l = 0$, implying that in the zero mass limit there is no helicity violation, as predicted earlier. We also find that

$$\frac{a_l}{b_l} \simeq -\frac{S_2^+}{S_1^+}. \tag{32}$$

It is straightforward to verify that $S_2^+/S_1^+ = T_2^+/T_1^+$, ensuring the consistency of the first two matching solutions. By means of the Bessel function relations (21) we see that

$$\frac{S_2^+}{S_1^+} \simeq \frac{\mp Z^2_{\pm(\nu+1)} J_l + Z^2_\nu J_{l+1}}{\mp Z^1_{\pm(\nu+1)} J_l + Z^1_\nu J_{l+1}}. \tag{33}$$

Making use, now, of the small argument form of Bessel functions, we find that b_l is suppressed by at least $(\omega R)^2$ with respect to a_l except for the case $0 > \nu > -1$ corresponding to $l = -1$, $\alpha_R \neq 0$ when the suppression is $(\omega R)^{2(1+\nu)}$. Hence, it is a good approximation to ignore b_l to leading order.

(i) $m \gg k$: We now have $B^+ \simeq B^- \simeq 1$ so, dropping the $\pm$, we can rewrite the first two matching solutions as

$$\begin{aligned} S\left(\tfrac{b_l}{a_l}\right) &+ (\lambda_l + S)\left(\tfrac{B_l}{a_l}\right) = -1, \\ T\left(\tfrac{b_l}{a_l}\right) &- (\lambda_l + T)\left(\tfrac{B_l}{a_l}\right) = -1, \end{aligned} \tag{34}$$

where $S = S_2/S_1$ and $T = T_2/T_1$. From these we see that if T dominates S then

$$\tfrac{a_l}{b_l} \simeq \tfrac{a_l}{B_l} \simeq -2S, \tag{35}$$

whilst if S dominates T

$$\tfrac{a_l}{b_l} \simeq -\tfrac{a_l}{B_l} \simeq -2T. \tag{36}$$

Investigation of S and T reveals that for $\nu > -1$, $T \gg S$, whilst for $-1 \leq \nu$, $S \gg T$. However, we need to know the magnitude of S and T in order to determine the relative suppression of b_l, and B_l. A summary of the results is

$T \gg S$:	$\nu \geq 0$:	$P \sim \frac{k^2}{\omega^2}(kR)^{-2(\nu+1)}$	Suppression of b_l, B_l greater than $(\omega R)^2$.
	$0 > \nu > -1$:	$P \sim (kR)^{-2\nu}$	b_l, B_l dominate a_l.
$S \gg T$:	$\nu = -1$:	$S \simeq \frac{2}{\pi}\log(kR)$	b_l, B_l relatively unsuppressed.
	$-1 > \nu$:	$S \sim \frac{k^2}{\omega^2}(kR)^{2\nu}$	Suppression of b_l, B_l greater than $(\omega R)^2$.

We note that, except for when $l = -1$, we can ignore the contributions of b_l and B_l. We now make use of this information to calculate the scattering cross-sections.

4 The Scattering Cross-Sections

The scattering amplitudes are given by the simple formulae

$$\left.\frac{d\sigma}{d\phi}\right|_{-}^{+} = \begin{cases} |f(\phi)|^2, & \text{helicity conserving cross-section.} \\ |g(\phi)|^2, & \text{helicity violating cross-section.} \end{cases}$$

where $f(\phi)$ and $g(\phi)$ are given by

$$f(\phi) = \sum_{l=-\infty}^{\infty} f_l e^{il\phi} \quad , \quad g(\phi) = \sum_{l=-\infty}^{\infty} g_l e^{il\phi}.$$

Ignoring the effect of b_l and B_l for the time being, we find that

$$\begin{aligned} f(\phi) &= -\frac{1}{\sqrt{2\pi ki}}\left[\sum_{l\geq 0}(1 - e^{i\pi\alpha_R})e^{-il(\pi-\alpha_R)} + \sum_{l\leq -1}(1 - e^{-i\pi\alpha_R})e^{il(\pi+\alpha_R)}\right] \\ &= \frac{ie^{-\frac{i\phi}{2}}}{\sqrt{2\pi ki}}\frac{\sin(\pi\alpha_R)}{\cos(\frac{\phi}{2})}, \\ g(\phi) &= 0, \end{aligned}$$

so the differential cross-sections for helicity conservation and helicity flip are

$$\left.\frac{d\sigma}{d\phi}\right|_{+} = \frac{1}{2\pi k}\frac{\sin^2(\pi\alpha_R)}{\cos^2(\frac{\phi}{2})}, \qquad \left.\frac{d\sigma}{d\phi}\right|_{-} = 0,$$

where we have denoted helicity conservation(flip) by $+(-)$. We recognise the first as the full Aharonov-Bohm cross-section. We now need to consider the effect of b_l and B_l for four different cases. Note that in each one, however, it is only the $l = -1$ mode which will contribute, confirming the work by Ganoulis[10].

(a) Non-integer α_R

(i) $m \ll k$

Since $B_l = 0$ there is no helicity flip in this case, as we predicted earlier by considering the zero mass limit of $[H, \underline{\Sigma}.\underline{\Pi}]$.

The effect of the b_l is to introduce a correction of order $(kR)^{2(1+\alpha_R)}$ such that

$$\left.\frac{d\sigma}{d\phi}\right|_{+} = \frac{1}{2\pi k}\frac{\sin^2(\pi\alpha_R)}{\cos^2(\frac{\phi}{2})}(1 + 2\mathrm{Re}(\Delta)) \tag{37}$$

where $\Delta = 2ie^{i(\pi\alpha_R-\phi)/2}\cos(\frac{\phi}{2})b_{-1}$.

(ii) $m \gg k$

Here we have the curious case where b_l and B_l dominate for the $l = -1$ mode, such that $b_l = B_l = e^{\frac{i\pi\nu}{2}}$. This leads to

$$\left.\frac{d\sigma}{d\phi}\right|_{+} = \frac{1}{2\pi k}\frac{\sin^2(\pi\alpha_R)}{\cos^2(\frac{\phi}{2})}|2+e^{-i\phi}|^2, \qquad \left.\frac{d\sigma}{d\phi}\right|_{-} = \frac{2}{\pi k}\sin^2(\pi\alpha_R),$$

so we have modified Aharonov-Bohm scattering, as predicted in [10].

(b) Integer α_R

(i) $m \ll k$

Since α is an integer (in our case zero) we see that the Aharonov-Bohm cross-section will vanish, and making use of our matching solution, we see that, to $\mathrm{O}(kR)^2$, even the correction due to b_l vanishes. Hence, there appears to be no scattering in this case. This is what we would expect, since if α and the mass are zero, there is essentially no interaction with the string, so we expect no scattering.

(ii) $m \gg k$

Once more, since α is an integer, the Aharonov-Bohm cross-section will vanish. However, for $m \gg k$, b_l and B_l are relatively unsuppressed and we obtain Everetts cross-section for both helicity conserving and helicity violating processes:

$$\left.\frac{d\sigma}{d\phi}\right|_{+} = \left.\frac{d\sigma}{d\phi}\right|_{-} = \frac{\pi}{8k}\frac{1}{[\log(kR)]^2} \tag{38}$$

Summary of Results

$\alpha_R \notin Z$	$m = 0$	Helicity conserving A-B cross-section. Zero helicity flip cross-section.
	$m \neq 0$	Modified A-B cross-section for both helicity conserving and flip processes.
$\alpha_R \in Z$	$m = 0$	No scattering.
	$m \neq 0$	Everett cross-section for both helicity conserving and flip processes.

5 Scattering from the Semi-local String

As mentioned in the introduction, we obtain our Lagrangian by modification of the Weinberg-Salaam model, taking SU(2) charge and gauge fields to be zero. Working with quarks, since they have non-integer hypercharge, and so may display an Aharonov-Bohm cross-section, we use a Lagrangian with Yukawa type couplings

$$\begin{aligned}\mathcal{L} &= i\bar{l}\gamma^\mu D^L_\mu l + i\bar{u}_R\gamma^\mu D^R_\mu u_R + i\bar{d}_R\gamma^\mu D^R_\mu d_R \\ &\quad - f^1_q(\bar{u}_L, \bar{d}_L)\tilde{\Phi} u_R + \text{h.c.} \\ &\quad - f^2_q(\bar{u}_L, \bar{d}_L)\Phi d_R + \text{h.c.}\end{aligned}$$

where $\tilde{\Phi} = i\tau_2\Phi$ and we assume that $f_q^{1,2}$ are real. It is seen that this is invariant under

$$\left.\begin{matrix} e_R \\ e_L \\ \Phi \end{matrix}\right\} \rightarrow U \left\{\begin{matrix} e_R \\ e_L \\ \Phi \end{matrix}\right. \quad \text{where } U = e^{i\beta(x)Y} \in U(1); \quad \text{local U(1) symmetry.}$$

$$\left.\begin{matrix} e_L \\ \Phi \end{matrix}\right\} \rightarrow S \left\{\begin{matrix} e_L \\ \Phi \end{matrix}\right. \quad \text{where } S \in SU(2); \qquad \text{global SU(2) symmetry.}$$

Note that hypercharge is conserved. Such a model has a string solution identical to that in the electroweak case, the only difference being that here we write B instead of Z for the relevant gauge field. This comes as no surprise as in both cases we are embedding the same Nielsen-Olesen vortex in a "larger" model.

Considering the case of the down-quark, a little algebra yields the equations of motion

$$\begin{aligned} i\gamma^\mu D^L_\mu d_L &- f^2_q f d_R e^{i\phi} = 0 \\ i\gamma^\mu D^R_\mu d_R &- f^2_q f d_L e^{-i\phi} = 0 \end{aligned}$$

which if we use the usual Dirac representation for the gamma matrices, write $d_L = (0, \psi)$,$d_R = (\chi, 0)$, restrict motion to the plane perpendicular to the z-axis, and set $\partial_t = -i\omega$ become

$$\begin{aligned} \omega\chi + i\sigma^j D^R_j\chi - f_e f e^{-i\phi}\psi &= 0 \\ \omega\psi - i\sigma^j D^L_j\psi - f_e f e^{i\phi}\chi &= 0 \end{aligned} \tag{39}$$

These are of the same form as (6) but with $\alpha_{L(R)}$ replaced by $Y_{L(R)}$. Since, in addition, $Y_L = Y_R + 1$, we can make use of the results previously calculated for the electroweak string.

For the down quark $Y_{d_R} = -\frac{2}{3} \notin Z$, so the dominant scattering is Aharonov-Bohm, and the cross-sections are identical to those for the electroweak string in the case of non-integer flux. Since our calculations were done assuming that $0 \geq \alpha_R > -1$ we can not immediately apply our results to the case of the electron, which has hypercharge $Y_{e_R} = -2$. However, we see that we should be able to modify our calculation by shunting the partial waves two along without physically altering the solution. Hence, we would expect the electron to display an Everett cross-section. The case of the up-quark is a little more tricky since we no longer have the relation $Y_L = Y_R + 1$ (in fact $Y_L = Y_R - 1$ here). This is not easily incorporated into our solution, and the most we can say is that, physically, we would expect the up quark to behave in a similar manner to the down-quark, since they both possess non-integer hypercharge, and display an Aharonov-Bohm interaction.

6 Conclusions

We have investigated elastic scattering off embedded defects like electroweak and semi-local strings, and, using the 'top-hat' model of the core, have been able to obtain a fuller solution to the scattering problem than previously found, whilst confirming existing results.

In the massive case, for fractional string flux, we have found a modified Aharonov-Bohm cross-section for both the helicity conserving and helicity-flip processes, whilst, for integer flux we have obtained the same cross-section as that of Everett [6], ie a strong interaction cross-section, but with logarithmic suppression. This latter case may be important for electron scattering from semi-local strings, whlist our techniques and results are also applicable to grand unified strings and chiral fermions. In the limit as the mass goes to zero, we see that helicity violating processes disappear, suggesting that such effects may be stronger at lower energies.

We stress that for fractional flux the total cross-section is a strong interaction cross-section, independent of the core radius. For integer flux the core radius appears in the logarithmic suppression factor, but this is only a mild dependence. Hence the evolution of these defects will be the same as that of local strings, with the strings strongly interacting with the surrounding plasma during the friction dominated era.

The significance of our result is that it clarifies the concept of Aharonov-Bohm phase for a particle whose internal degrees of freedom couple to a flux tube, or string (in our example in a left-right asymmetric way). Hence, if cosmic strings are ever found it will be possible to study their interactions with matter and maybe even "probe" their core through this type of scattering experiment. Our results may also have implications for grand unified strings. This is currently under investigation [13].

One of us (A.P.M.) acknowledges S.E.R.C. for financial support.

Appendix: Matching at $r = R$

Our solution for $r < R$ is

$$\begin{pmatrix} c_l J_l e^{il\phi}(\omega r) \\ ic_l J_{l+1} e^{i(l+1)\phi}(\omega r) \\ d_l J_{l+1} e^{i(l+1)\phi}(\omega r) \\ id_l J_{l+2} e^{i(l+2)\phi}(\omega r) \end{pmatrix}$$

whilst our external solution is

$$\begin{pmatrix} (a_l Z^1_\nu(kR) & + & b_l Z^2_\nu(kR) & + & A_l Z^1_\nu(kR) & + & B_l Z^2_\nu(kR))e^{il\phi} \\ i(a_l Z^1_{(\nu+1)}(kR) & + & b_l Z^2_{(\nu+1)}(kR) & - & A_l Z^1_{(\nu+1)}(kR) & - & B_l Z^2_{(\nu+1)}(kR))e^{i(l+1)\phi} \\ (a_l B^+ Z^1_\nu(kR) & + & b_l B^+ Z^2_\nu(kR) & + & A_l B^- Z^1_\nu(kR) & + & B_l B^- Z^2_\nu(kR))e^{i(l+1)\phi} \\ i(a_l B^+ Z^1_{(\nu+1)}(kR) & + & b_l B^+ Z^2_{(\nu+1)}(kR) & - & A_l B^- Z^1_{(\nu+1)}(kR) & - & B_l B^- Z^2_{(\nu+1)}(kR))e^{i(l+2)\phi} \end{pmatrix}$$

so by continuity

$$\begin{array}{rcrcrcrcl} Z^1_\nu a_l & + & Z^2_\nu b_l & + & Z^1_\nu A_l & + & Z^2_\nu B_l & = & J_l c_l \\ Z^1_{(\nu+1)} a_l & + & Z^2_{(\nu+1)} b_l & - & Z^1_{(\nu+1)} A_l & - & Z^2_{(\nu+1)} B_l & = & J_{l+1} c_l \\ B^+ Z^1_\nu a_l & + & B^+ Z^2_\nu b_l & + & B^- Z^1_\nu A_l & + & B^- Z^2_\nu B_l & = & J_{l+1} d_l \\ B^+ Z^1_{(\nu+1)} a_l & + & B^+ Z^2_{(\nu+1)} b_l & - & B^- Z^1_{(\nu+1)} A_l & - & B^- Z^2_{(\nu+1)} B_l & = & J_{l+2} d_l \end{array}$$

whilst the first derivatives give us

$$\begin{array}{rrrrcr} Z^{1'}_\nu a_l + & Z^{2'}_\nu b_l + & Z^{1'}_\nu A_l + & Z^{2'}_\nu B_l & = & (J'_l - \frac{\alpha_R}{R} J_l) c_l + \quad m J_{l+2} d_l \\ Z^{1'}_{(\nu+1)} a_l + & Z^{2'}_{(\nu+1)} b_l - & Z^{1'}_{(\nu+1)} A_l - & Z^{2'}_{(\nu+1)} B_l & = & (J'_{l+1} + \frac{\alpha_R}{R} J_{l+1}) c_l - \quad m J_{l+1} d_l \\ B^+ Z^{1'}_\nu a_l + & B^+ Z^{2'}_\nu b_l - & B^- Z^{1'}_\nu A_l + & B^- Z^{2'}_\nu B_l & = & -m J_{l+1} c_l + (J'_{l+1} - \frac{\alpha_L}{R} J_{l+1}) d_l \\ B^+ Z^{1'}_{(\nu+1)} a_l + & B^+ Z^{2'}_{(\nu+1)} b_l - & B^- Z^{1'}_{(\nu+1)} A_l - & B^- Z^{2'}_{(\nu+1)} B_l & = & m J_l c_l + (J'_{l+2} + \frac{\alpha_L}{R} J_{l+2}) d_l \end{array}$$

Note that here $'$ denotes $\frac{d}{dr}$. We now make use of the fact that $1/B^+ = B^-$ to rewrite our matching conditions as

$$\begin{array}{rcrcrcl} S^+_1 a_l & + & S^+_2 b_l & + & (\lambda_l S^-_1 + S^-_2) B_l & = & 0 \\ T^+_1 a_l & + & T^+_2 b_l & - & (\lambda_l T^-_1 + T^-_2) B_l & = & 0 \\ B^+ U^+_1 a_l & + & B^+ U^+_2 b_l & + & B^-(\lambda_l U^-_1 + U^-_2) B_l & = & 0 \\ B^+ V^+_1 a_l & + & B^+ V^+_2 b_l & - & B^-(\lambda_l V^-_1 + V^-_2) B_l & = & 0 \end{array}$$

where

$$\begin{array}{rcrcl} S^\pm_{1,2} & = & Z^{1,2'}_\nu & - & (\frac{J'_l}{J_l} - \frac{\alpha_R}{R} + m B^\pm \frac{J_{l+2}}{J_{l+1}}) Z^{1,2}_\nu \\ T^\pm_{1,2} & = & Z^{1,2'}_{(\nu+1)} & - & (\frac{J'_{l+1}}{J_{l+1}} + \frac{\alpha_R}{R} - m B^\pm \frac{J_{l+1}}{J_{l+2}}) Z^{1,2}_{(\nu+1)} \\ U^\pm_{1,2} & = & Z^{1,2'}_\nu & - & (\frac{J'_{l+1}}{J_{l+1}} - \frac{\alpha_L}{R} - m B^\mp \frac{J_{l+1}}{J_l}) Z^{1,2}_\nu \\ V^\pm_{1,2} & = & Z^{1,2'}_{(\nu+1)} & - & (\frac{J'_{l+2}}{J_{l+2}} + \frac{\alpha_L}{R} + m B^\mp \frac{J_{l+l}}{J_{l+1}}) Z^{1,2}_{(\nu+1)} \end{array}$$

References

[1] A.Vilenkin *Physics Reports* **121**(1985), 263

[2] R.D.Peccei and H.R.Quinn *Phys.Rev.Lett.* **38**(1977), 1440 *Phys.Rev.* **D16**(1977), 1791

[3] H.B.Nielsen and P.Olesen *Nuc.Phys.* **B61**(1973), 45

[4] M.G.Alford and F.Wilczek *Phys.Rev.Lett.* **62**(1989), 1071

[5] W.B.Perkins, L.Perivolaropoulos, A.C.Davis, R.H.Brandenberger and A.Matheson *Nuc.Phys.* **B353**(1991), 237

[6] A.E.Everett *Phys.Rev.D* **24**(1981), 858

[7] T.Vachaspati and A.Achúcarro *Phys.Rev.D* **44**(1991), 3067

[8] M.Hindmarsh *Phys.Rev.Lett.* **68**(1992), 1263

[9] T.Vachaspati *Phys.Rev.Lett.* **68**(1992), 1977 **35**(1987), 1138

[10] N.Ganoulis *Phys.Lett.* **B298**(1993), 63

[11] T.W.B.Kibble *Physics Reports* **67**(1980), 183

[12] M.James, L.Perivolaropoulos and T.Vachaspati *Phys.Rev.D* **46**(1992), 5232, Nuc.Phys. **B395**(1993), 534

[13] A.C.Davis and R.Jeannerot *DAMTP preprint (in preparation)*